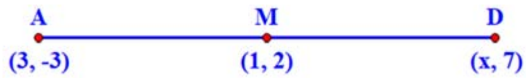
# Formal Geometry    Ch 1 Review Worksheet

1. The midpoint of a line segment AD is (1, 2). Point A has coordinates of (3, -3) and point D coordinates are ($x$, 7). Find the value of $x$.

Name:______________________________

It is helpful to use a line diagram when dealing with midpoint problems. Label the endpoints and midpoint, and identify the coordinates of each:




The difference between points **A** and **M** can be expressed in two dimensions as a vector using "⟨ ⟩" instead of "( )". Let's find the difference (note: "difference" implies subtraction).

$$\begin{array}{rl} (1, \quad 2) & \text{Point } \mathbf{M} \\ -\ (3, -3) & \text{Point } \mathbf{A} \\ \hline \langle -2, \quad 5 \rangle & \text{Difference vector} \end{array}$$

The difference vector can then be applied to the midpoint to get the coordinates of point **D**.

$$\begin{array}{rl} (\ \ 1, 2) & \text{Point } \mathbf{M} \\ +\ \langle -2, 5 \rangle & \text{Difference vector} \\ \hline (-\mathbf{1}, 7) & \text{Point } \mathbf{D}\text{. Therefore, we conclude that } \boldsymbol{x = -1}. \end{array}$$

Note that the $y$-value of point **D** in the solution matches the $y$-value of point **D** in the statement of the problem.

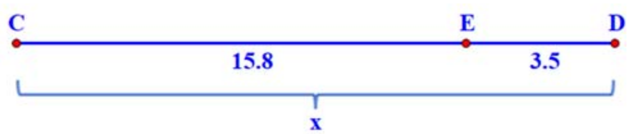
2. Find two possible lengths for $\overline{CD}$ if C, D, and E are collinear, CE = 15.8 cm, and DE = 3.5 cm.

There are two possible line diagrams for this problem: 1) **D** is between **C** and **E**, 2) **E** is between **C** and **D**. In these diagrams, we show distances instead of point values:

<u>Case 1</u>


$x = 15.8 - 3.5 = \mathbf{12.3\ cm}$

<u>Case 2</u>




$x = 15.8 + 3.5 = \mathbf{19.3\ cm}$

3. Find the length of $\overline{RT}$ if S is between R and T, S is a midpoint, RS = $3x + 3$, and ST = $5x - 6$.

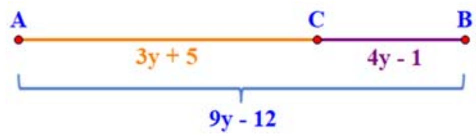

Since **S** is a midpoint, $3x + 3 = 5x - 6$

$$9 = 2x$$

$$\frac{9}{2} = x$$

Finally, $\mathbf{RT} = (3x + 3) + (5x - 6) = 8x - 3 = 8\left(\frac{9}{2}\right) - 3 = \mathbf{33}$

4. Find the value of $y$ if AC = $3y + 5$, CB = $4y - 1$, AB = $9y - 12$, and point C lies between A and B.

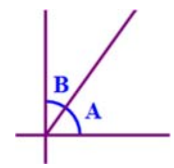



Based on the diagram, we have: $(3y+5)+(4y-1)=9y-12$

$$7y+4=9y-12$$
$$16=2y$$
$$\mathbf{8}=\boldsymbol{y}$$

5. Two angles are complementary. The measure of one angle is 21 more than twice the measure of the other angle. Find the measures of the angles.

Let the two angles be called angle $\mathbf{A}$ and angle $\mathbf{B}$. Let's rewrite the problem in terms of these two angles.




"Angles $\mathbf{A}$ and $\mathbf{B}$ are complementary. $m\angle\mathbf{A} = 21° + 2(m\angle\mathbf{B})$."

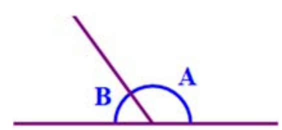
Let the measures of the angles be represented by the names of the angles. Then,

$$\left.\begin{aligned} A+B&=90° \\ A&=21°+2B \end{aligned}\right\} \longrightarrow \begin{aligned} 2A+2B&=180° \\ +\quad A-2B&=\ \ 21° \\ \hline 3A\qquad&=201° \\ A\qquad&=\mathbf{67°} \end{aligned} \qquad \begin{aligned} A+B&=90° \\ 67°+B&=90° \\ \hline \boldsymbol{B}&=\mathbf{23°} \end{aligned}$$

The measures of the two angles then, are, $\mathbf{67}°$ and $\mathbf{23}°$

6. If a supplement of an angle has a measure 78 less than the measure of the angle, what are the measures of the angles?

The two angles are supplementary. Let's call them angle $\mathbf{A}$ and angle $\mathbf{B}$. Rewrite the problem in terms of these two angles.



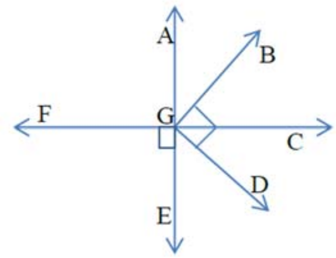

"Angles $\mathbf{A}$ and $\mathbf{B}$ are supplementary. $m\angle\mathbf{A} - \mathbf{78°} = m\angle\mathbf{B}$."

Let the measures of the angles be represented by the names of the angles. Then,

$$\left.\begin{aligned} A+B\quad&=180° \\ A-78°&=B \end{aligned}\right\} \longrightarrow \begin{aligned} A+B&=180° \\ +\quad A-B&=\ \ 78° \\ \hline 2A\qquad&=258° \\ A\qquad&=\mathbf{129°} \end{aligned} \qquad \begin{aligned} A+B&=180° \\ 129°+B&=180° \\ \hline \boldsymbol{B}&=\ \mathbf{51°} \end{aligned}$$

The measures of the two angles then, are, $\mathbf{129}°$ and $\mathbf{51}°$

***For # 7-8 use the figure at the right.***



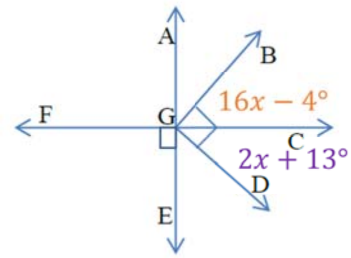

7. If $m\angle FGE = 5x + 10$, find the value of $x$ so that $\overleftrightarrow{FC} \perp \overleftrightarrow{AE}$.

$FC \perp AE$ implies that $\angle FGE$ is a right angle (i.e., $m\angle FGE = 90°$).

Then, $5x + 10° = 90°$

$5x = 80°$

$\boldsymbol{x = 16°}$




8. If $m\angle BGC = 16x - 4$ and $m\angle CGD = 2x + 13$, find the value of $x$ so That $\angle BGD$ is a right angle.

$\angle BGD$ is a right angle (i.e., $m\angle BGD = 90°$).

Then, $(16x - 4°) + (2x + 13°) = 90°$

$18x + 9° = 90°$

$18x = 81°$

$\boldsymbol{x = 4.5°}$

***For exercises 9-12 find the distance between each pair of points.***

9. A(0, 0), B (6, 8)

The formula for the distance between points is: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let point 1 be $A(0, 0)$, and let point 2 be $B(6, 8)$. Then,

$\boldsymbol{d} = \sqrt{(6-0)^2 + (8-0)^2} = \sqrt{100} = \mathbf{10}$ Note: 6-8-10 is a Pythagorean Triple

10. R(-2, 3), S(3, 15)

$\boldsymbol{d} = \sqrt{\left(3 - (-2)\right)^2 + (15-3)^2} = \sqrt{169} = \mathbf{13}$ Note: 5-12-13 is a Pythagorean Triple

11. K(1, -2), L(9, 13)

$\boldsymbol{d} = \sqrt{(9-1)^2 + \left(13 - (-2)\right)^2} = \sqrt{289} = \mathbf{17}$ Note: 8-15-17 is a Pythagorean Triple

12. E(-12, 2), F(-9, 6)

$\boldsymbol{d} = \sqrt{\left(-9 - (-12)\right)^2 + (6-2)^2} = \sqrt{25} = \mathbf{5}$ Note: 3-4-5 is a Pythagorean Triple

***For exercises 13-14 find the coordinates of the midpoint of a segment with the given endpoints.***

13. K(-9, 3), H(5, 7)

The coordinates of the midpoint are the averages of the coordinates of the endpoints.

$$\begin{array}{rl} (-9,\ \ 3) & \text{Point } \mathbf{K} \\ +\,(5,\ \ 7) & \text{Point } \mathbf{H} \\ \hline (-4, 10) \div 2 = (-\mathbf{2}, \mathbf{5}) & \textbf{Midpoint} \end{array}$$

14. W(-12, -7), T(-8, -4)

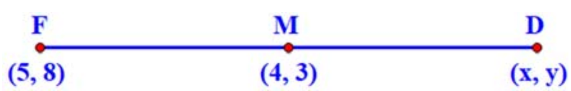
The coordinates of the midpoint are the averages of the coordinates of the endpoints.

$$\begin{array}{rl} (-12, -7) & \text{Point } \mathbf{W} \\ +\,(-8, -4) & \text{Point } \mathbf{T} \\ \hline (-20, -11) \div 2 = (-\mathbf{10}, -\mathbf{5.5}) & \textbf{Midpoint} \end{array}$$

***For exercises 15-16 find the coordinates of the missing endpoint if M is the midpoint of $\overline{DF}$.***

15. F(5, 8), M(4, 3)

This problem is similar to Problem 1:




The difference between points $\mathbf{F}$ and $\mathbf{M}$ can be expressed in two dimensions as a vector using "⟨ ⟩" instead of "( )". Let's find the difference (note: "difference" implies subtraction).

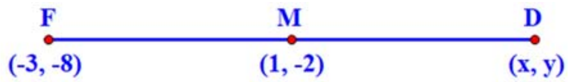
$$\begin{array}{rl} (\ \ 4,\ \ 3) & \text{Point } \mathbf{M} \\ -\,(\ \ 5,\ \ 8) & \text{Point } \mathbf{F} \\ \hline \langle -1, -5 \rangle & \text{Difference vector} \end{array}$$

The difference vector can then be applied to the midpoint to get the coordinates of point $\mathbf{D}$.

$$\begin{array}{rl} (\ \ 4,\ \ 3) & \text{Point } \mathbf{M} \\ +\,\langle -1, -5 \rangle & \text{Difference vector} \\ \hline (\ \ \mathbf{3}, -\mathbf{2}) & \textbf{Point D} \end{array}$$

16. F(-3, -8), M(1, -2)

This problem is similar to Problem 1:



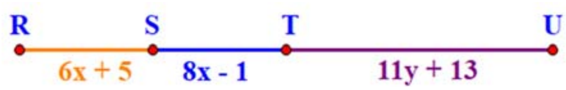

The difference between points **F** and **M** can be expressed in two dimensions as a vector using "⟨ ⟩" instead of "( )". Let's find the difference (note: "difference" implies subtraction).

$$\begin{array}{rl} (\ \ 1, -2) & \text{Point } \mathbf{M} \\ -\,(-3, -8) & \text{Point } \mathbf{F} \\ \hline \langle\ \ 4,\ \ \ 6\rangle & \text{Difference vector} \end{array}$$

The difference vector can then be applied to the midpoint to get the coordinates of point **D**.

$$\begin{array}{rl} (\ \ 1, -2) & \text{Point } \mathbf{M} \\ +\,\langle\ \ 4,\ \ \ 6\rangle & \text{Difference vector} \\ \hline (\ \ \mathbf{5},\ \ \ \mathbf{4}) & \textbf{Point D} \end{array}$$

17. Find the value of $y$ if S is the midpoint of $\overline{RT}$ , T is the midpoint of $\overline{RU}$ RS = $6x + 5$, ST = $8x - 1$, and TU = $11y + 13$.




Since $S$ is the Midpoint of $\overline{RT}$, $6x + 5 = 8x - 1$

$$6 = 2x$$
$$3 = x$$

Also, $RT = (6x + 5) + (8x - 1) = 14x + 4 = 14(3) + 4 = 46$

Since $T$ is the Midpoint of $\overline{RU}$, $RT = TU$, so $TU = 11y + 13 = 46$

$$11y = 33$$
$$\boldsymbol{y = 3}$$

18. Find all of the values of $x$ that will make $\angle A$ an obtuse angle, given $m\angle A = 12x - 6$.

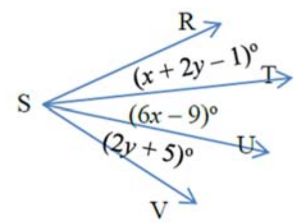
An obtuse angle has a measure greater than 90° and less than 180°. So,

$90° < 12x - 6° < 180°$

$96° < \quad 12x \quad < 186°$ after adding 6° to all three parts of the inequality

$\mathbf{8° < \quad \boldsymbol{x} \quad < 15.5°}$ after dividing all three parts of the inequality by 12

19. Find $m\angle RST$ if $\overrightarrow{ST}$ bisects $\angle RSU$ and $\overrightarrow{SU}$ bisects $\angle TSV$.




Since $\overrightarrow{ST}$ bisects $\angle RSU$, we have: $x + 2y - 1 = 6x - 9$

Since $\overrightarrow{SU}$ bisects $\angle TSV$, we have: $6x - 9 = 2y + 5$

There are a number of ways to solve these simultaneous equations. Let's use substitution:

Both equations have an expression equal to $6x - 9$, so let's set these expressions equal to each other:

$x + 2y - 1 = 2y + 5$

$x - 1 = 5$ after subtracting $2y$ from both sides

$x = 6$

Use the second equation above to find $y$:

$6x - 9 = 2y + 5$

$6(6) - 9 = 2y + 5$

$27 = 2y + 5$

$22 = 2y$

$y = 11$

Finally: $\boldsymbol{m\angle RST} = (x + 2y - 1)° = (6 + 2(11) - 1)° = \mathbf{27}°$

20. Find $m\angle 1$ if $\angle 1$ is complementary to $\angle 2$, $\angle 2$ is supplementary to $\angle 3$, and $m\angle 3 = 126$.

Let's turn this into equations because the English is confusing.

$m\angle 1 + m\angle 2 = 90°$

$m\angle 2 + m\angle 3 = 180°$

$m\angle 3 = 126°$

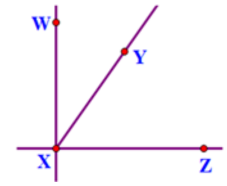
Working with these equations from bottom to top, we get:

$m\angle 3 = 126°$

$m\angle 2 + m\angle 3 = m\angle 2 + 126° = 180°$, so $m\angle 2 = 54°$

$m\angle 1 + m\angle 2 = m\angle 1 + 54° = 90°$ so $\boldsymbol{m\angle 1} = \mathbf{36}°$

21. Find the value of $y$ if $\overrightarrow{XW} \perp \overrightarrow{XZ}$, Y is in the interior of $\angle WXZ$, $m\angle WXY = 6y - 3$, and $m\angle YXZ = 4y + 13$.




Sometimes, it helps to draw your own picture. From the picture shown to the right, it is easy to see how the angles line up. We have:

$m\angle WXY + m\angle YXZ = 90°$ because the two angles must be complementary. So,

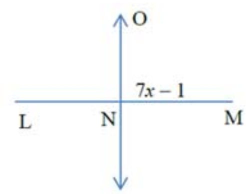
$(6y - 3°) + (4y + 13°) = 90°$

$10y + 10° = 90°$

$10y = 80°$

$\boldsymbol{y = 8°}$

22. Find the length of $\overline{LM}$ if $\overleftrightarrow{ON}$ is the bisector of $\overline{LM}$ and $LN = 3x + 2$.




Since $\overleftrightarrow{ON}$ bisects $\overline{LM}$, the segments $\overline{LN}$ and $\overline{NM}$ are equal in length. So,

$3x + 2 = 7x - 1$

$3 = 4x$

$x = \frac{3}{4}$

Then, $LM = LN + NM = (3x + 2) + (7x - 1)$

$= 10x + 1$

$= 10\left(\frac{3}{4}\right) + 1$

$= \frac{30}{4} + \frac{4}{4} = \frac{34}{4} = \boldsymbol{\frac{17}{2}} = \boldsymbol{8.5}$

23. Find the measure of an angle and its complement if one of the angle measures 24 degrees more than the other.

We have complementary angles with measures $x$ and $y$ such that the following are true:

$$\left.\begin{aligned} x + y &= 90° \\ x &= y + 24° \end{aligned}\right\} \longrightarrow \quad \begin{aligned} x &= -y + 90° \\ + \quad x &= \phantom{-}y + 24° \\ \hline 2x &= 114° \\ \boldsymbol{x} &= \boldsymbol{57°} \end{aligned} \quad \longrightarrow \quad \begin{aligned} x + y &= 90° \\ 57° + y &= 90° \\ \boldsymbol{y} &= \boldsymbol{33°} \end{aligned}$$

So, the measures of the two angles are $\boldsymbol{57°}$ and $\boldsymbol{33°}$.

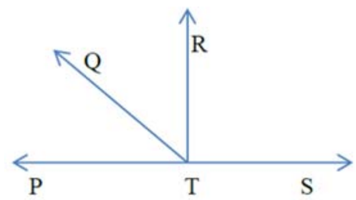
24. The measure of the supplement of an angle is 36 less than the measure of the angle. Find the measures of the angles.

We have supplementary angles with measures $x$ and $y$ such that the following are true:

$$\left.\begin{aligned} x + y &= 180^\circ \\ x &= y - 36^\circ \end{aligned}\right\} \longrightarrow \begin{aligned} x &= -y + 180^\circ \\ + \quad x &= \phantom{-}y - 36^\circ \\ \hline 2x &= 144^\circ \\ \boldsymbol{x} &= \mathbf{72^\circ} \end{aligned} \longrightarrow \begin{aligned} x + y &= 180^\circ \\ 72^\circ + y &= 180^\circ \\ \boldsymbol{y} &= \mathbf{108^\circ} \end{aligned}$$

So, the measures of the two angles are $\mathbf{72^\circ}$ and $\mathbf{108^\circ}$.

***For exercises 25-26 use the figure at the right.***




25. $If\ m\angle RTS = 8x + 18$, find the value of $x$ so that $\overrightarrow{TR} \perp \overrightarrow{TS}$.

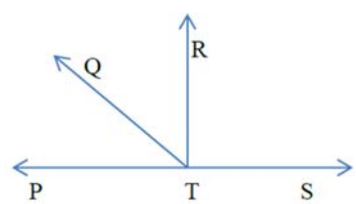
$\overrightarrow{TR}$ and $\overrightarrow{TS}$ will be perpendicular if $m\angle RTS = 90^\circ$. So,

$8x + 18^\circ = 90^\circ$

$8x = 72^\circ$

$\boldsymbol{x} = \mathbf{9^\circ}$

26. If $m\angle\mathrm{PTQ} = 3y - 10$ and $\mathrm{m}\angle\mathrm{QTR} = y$, find the value of $y$ so that $\angle\mathrm{PTR}$ is a right angle.




If $\angle PTR$ is a right angle, then, $\angle PTQ$ and $\angle QTR$ are complementary. So,

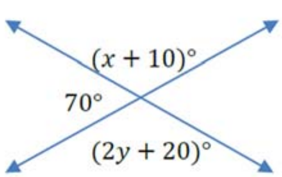
$(3y - 10^\circ) + y = 90^\circ$

$4y - 10^\circ = 90^\circ$

$4y = 100^\circ$

$\boldsymbol{y} = \mathbf{25^\circ}$

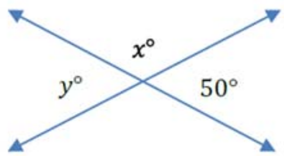
27. Solve for x and y.




We have two pair of supplementary angles that can be solved as follows:

| | |
|---|---|
| $(x + 10)^\circ + 70^\circ = 180^\circ$ | $(2y + 20)^\circ + 70^\circ = 180^\circ$ |
| $x + 80^\circ = 180^\circ$ | $2y + 90^\circ = 180^\circ$ |
| $\boldsymbol{x} = \mathbf{100^\circ}$ | $2y = 90^\circ$ |
| | $\boldsymbol{y} = \mathbf{45^\circ}$ |

28. By how much does x exceed y?




This question is asking for "$x - y$". Be careful!

As in the previous problem, we have two pair of supplementary angles that can be solved as follows:

| | |
|---|---|
| $x + 50 = 180$ | $130 + y = 180$ |
| $x = 130$ | $y = 50$ |

$\boldsymbol{x - y} = 130 - 50 = \mathbf{80}$

Note that another way to find $y$ is that the angles shown as $y°$ and $50°$ are vertical angles, so they must have the same measure. Therefore, $y = 50$.

**Problems 29 to 34 are Algebra Review**

29) Factor: $x^2 + 8x + 15$

To factor a trinomial with a lead coefficient of 1, consider the following (see p. 68 in the Algebra Handbook available on www.mathguy.us).

$$(\boldsymbol{x}+\boldsymbol{p})\cdot(\boldsymbol{x}+\boldsymbol{q}) = \boldsymbol{x}^2 \underset{\uparrow\,\text{sign 1}}{+} \underbrace{(\boldsymbol{p}+\boldsymbol{q})}_{\text{coefficient of x}}\boldsymbol{x} \underset{\uparrow\,\text{sign 2}}{+} \underbrace{(\boldsymbol{pq})}_{\text{constant}}$$

For this problem, $p + q = 8$, and $p \cdot q = 15$.

Thinking about possibilities we conclude that $p$ and $q$ must be 3 and 5. Then,

$$x^2 + 8x + 15 = (\boldsymbol{x} + \mathbf{3})(\boldsymbol{x} + \mathbf{5})$$

30) Factor: $x^2 - 9x + 14$

For this problem, $p + q = -9$, and $p \cdot q = 14$.

$p$ and $q$ must both be negative because the coefficient of $x$ in the original expression is negative, but the constant term is positive.

Thinking about possibilities we conclude that $p$ and $q$ must be $-2$ and $-7$. Then,

$$x^2 - 9x + 14 = (\boldsymbol{x} - \mathbf{2})(\boldsymbol{x} - \mathbf{7})$$

31) Factor: $-12x^2 - 26x + 10$

First, factor out the greatest common factor: $-12x^2 - 26x + 10 = -2(6x^2 + 13x - 5)$

To factor a trinomial with a lead coefficient other than 1, consider the steps shown on p. 69 in the Algebra Handbook. *The AC Method works without considering multiple possible solutions.*

Alternatively, the factored form must be: $(mx + p)(nx + q)$. So, $(m \cdot n)$ is the coefficient of $x^2$ and $(p \cdot q)$ is the constant term. If there are not many possibilities for $m, n, p, q$, we can try various combinations of $m, n, p, q$ to see if the correct coefficient of the $x$ term results when multiplying $(mx + p)(nx + q)$. Doing this with $(6x^2 + 13x - 5)$ from above, we settle on:

$$-12x^2 - 26x + 10 \;=\; -2(6x^2 + 13x - 5) \;=\; \mathbf{-2}(\mathbf{2}\boldsymbol{x} + \mathbf{5})(\mathbf{3}\boldsymbol{x} - \mathbf{1}).$$

32) Multiply: $(4x-3)^2$

Page 62 of the Algebra Handbook describes the FOIL and Box methods of multiplying polynomials. Let's use FOIL for this problem.

Multiply: $(4x-3)(4x-3)$

First: $4x \cdot 4x = 16x^2$

Outside: $4x \cdot (-3) = -12x$

Inside: $(-3) \cdot 4x = -12x$

Last: $(-3) \cdot (-3) = 9$

Now, add the resulting terms: $16x^2 - 12x - 12x + 9 = \mathbf{16x^2 - 24x + 9}$

33) Multiply: $(x+5)^2$

Multiply: $(x+5)(x+5)$

First: $4 \cdot 4 = x^2$

Outside: $x \cdot 5 = 5x$

Inside: $5 \cdot x = 5x$

Last: $5 \cdot 5 = 25$

Now, add the resulting terms: $x^2 + 5x + 5x + 25 = \mathbf{x^2 + 10x + 25}$

34) Multiply: $(3\sqrt{6})^2$

$$(3\sqrt{6})^2 = 3^2 \cdot (\sqrt{6})^2 = 9 \cdot 6 = \mathbf{54}$$

Alternatively,

$$(3\sqrt{6}) \cdot (3\sqrt{6}) = 3 \cdot 3 \cdot \sqrt{6} \cdot \sqrt{6} = 9 \cdot 6 = \mathbf{54}$$

35) What is the midpoint of the segment of $3y = 4x + 15$ between $x = 6$ *and* $x = 21$ ?

When $x = 6$, $3y = 4 \cdot 6 + 15 = 39$, so $y = 13$. The resulting point is $(6, 13)$.

When $x = 21$, $3y = 4 \cdot 21 + 15 = 99$, so $y = 33$. The resulting point is $(21, 33)$.

The coordinates of the midpoint are the averages of the coordinates of the endpoints.

| | |
|---|---|
| $(\ 6, 13)$ | 1st Point |
| $+\ (21, 33)$ | 2nd Point |
| $(27, 46) \div 2 = \mathbf{(13.5, 23)}$ **Midpoint** | |